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UNSTABLE PLANETARY CONFIGURATIONS

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THE present article describes a new type of planetary instability which arises in connexion with the theory of the internal structure of the earth which I have developed[1]. I attribute the sharp core discontinuity at about half-way to the centre of the earth, at which the density almost doubles[2], to a phase transition under pressure. This is in contrast to the older view that the core is iron which has separated under gravity from the less dense silicates of the surrounding mantle. In the new theory, the enormous pressure of 1·4 million atmospheres at the boundary of the core is presumed to have partially destroyed the molecular structure of olivine, the principal constituent of the mantle. The olivine in the mantle has the familiar ionic structure; in the core it is in a dense metallic state. In an ordinary polymorphic transition, in which the molecules are merely rearranged into a different crystal structure, the density changes by only a few per cent. But in a transition to a metallic phase, in which the molecules themselves are partially destroyed, the increase in density will be of the order of 100 per cent.

$(Fe, Mg)_2SiO_4$
dens ≈ 3.35

The transition of a non-metal to a metallic phase is not a common occurrence in the laboratory. Usually a pressure of the order of a million atmospheres is necessary to make a non-metal metallic, and the highest pressure so far attained is only 100,000 atmospheres. But both metallic and non-metallic modifications of a few elements exist under laboratory conditions; examples are white and grey tin, metallic and yellow arsenic, black and yellow phosphorus. The metallic modification is always considerably denser; metallic arsenic is more than 2·5 times as dense as the yellow allotrope. The new interpretation of the core in terms of a phase transition is compatible with all seismic evidence of the earth's internal structure, and it accounts for some features not previously explained.

d = 2.0
d = 5.73
d = 7.28
d = 5.75
d = 1.82
d = 2.70

The new theory receives considerable support from the observed mean densities of the terrestrial planets. Jeffreys[3] has shown that, on the iron core hypothesis, the terrestrial planets have different chemical compositions; the smaller planets have relatively smaller cores, and so smaller proportions of iron.

This is difficult, if not impossible, to reconcile with the planets having a common origin. The difficulty is removed by the new interpretation of the core. The new theory, together with the assumptions current in geophysics, implies that the variation in density within the earth is due mainly to the influence of pressure and not to changes in chemical composition. It is true that the crust contains a number of chemically distinct layers, but they account for only one per cent of the total mass. The pressure–density relationship for the material of the earth, which can be derived from seismic data[2], has been used to compute the mean densities of smaller planets of similar composition. It is found that all the terrestrial planets have the same chemical composition[1]. The smaller planets have smaller cores because of their smaller central pressures; and in the smallest planets, Mars and Mercury, the pressures are too low to support a core.

In the course of these calculations on the terrestrial planets, it was found that the smallest planetary cores are unstable and so would not occur in Nature. This feature is a direct consequence of the main assumption underlying the calculations, namely, that the jump in density at the boundary of the core is controlled by pressure. This phenomenon, which has recently been examined in some detail by Lighthill[4] and myself[5], may have played a significant part in the evolution of the solar system. To illustrate the effect, I have made calculations[5] on planets composed of a material which has a constant density ρ_0 at all pressures below a critical pressure p_c, and a constant density $\lambda\rho_0$, which is greater than ρ_0, at all higher pressures; in other words, both phases of the material are assumed to be incompressible. The central pressure always specifies the planetary configuration unambiguously; that is, the central pressure determines uniquely the mass and radius of the planet and the radius of its core. The radius of the core is necessarily an increasing function of the central pressure. But, for very small cores, the mass and radius of the planet are decreasing functions of the central pressure if the density jump at the boundary of the core exceeds 50 per cent (that is, if $\lambda > \frac{3}{2}$). The relationship between the mass M of the planet and the central pressure is illustrated in Fig. 1; the behaviour of the total radius is qualitatively similar. The mass M is a continually increasing function of the central pressure if the parameter λ does not exceed $\frac{3}{2}$; any planet of given mass has therefore only one possible equilibrium configuration. But if λ exceeds $\frac{3}{2}$, a mass M in a certain critical range

$$M_1 < M < M_0 \qquad (1)$$

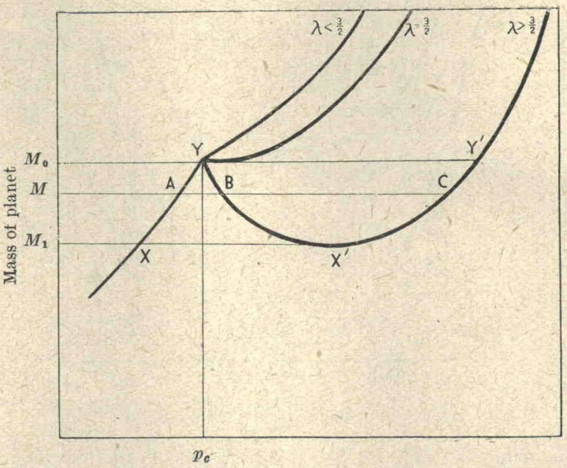

Fig. 1. Behaviour of the mass M of a planet as a function of the central pressure

is compatible with three different planetary configurations, denoted by A, B and C in the diagram. Configuration A has no core, and B has a smaller core than C. In general, the three configurations have different total radii and different degrees of stability.

Lighthill[4] has extended these considerations to the general case of planets composed of a material the density of which is a continuous function of the pressure up to a critical pressure p_c, and at the critical pressure the density is assumed to jump discontinuously from ρ_0 to $\lambda\rho_0$. Lighthill has shown that the derivative $(\mathrm{d}M/\mathrm{d}P)$ of the mass of the planet with respect to the central pressure P changes discontinuously by a factor

$$(3-2\lambda)/\lambda^2 \qquad (2)$$

when the central pressure is equal to the critical pressure p_c. The derivative with respect to P of any other bulk characteristic, such as the radius of the planet, changes by the same factor. This factor is positive if λ does not exceed $\frac{3}{2}$, and so the mass M is a continually increasing function of the central pressure as shown in Fig. 1. But the factor (2) is negative if λ exceeds $\frac{3}{2}$, so that $(\mathrm{d}M/\mathrm{d}P)$ changes sign. As the mass M cannot decrease with P indefinitely, the behaviour must be qualitatively as shown in Fig. 1. The earlier considerations therefore apply also in the general case of a compressible

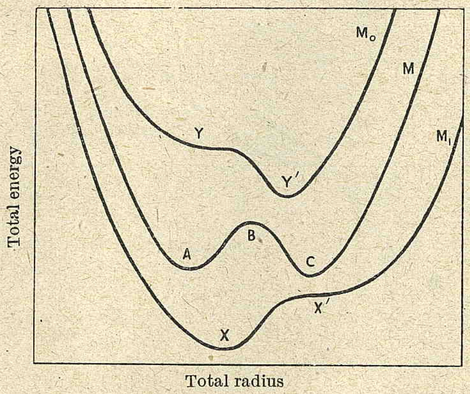

Fig. 2. Energy of a planet as a function of its total radius

material. Using Bullen's[2] estimate of the density distribution within the earth, it is found by numerical integration that the planet with its centre at the critical pressure has the following mass and radius :

$$M_0 = 0{\cdot}80\, M_E, \quad R_0 = 6{,}300 \text{ km.}, \qquad (3)$$

M_E being the mass of the earth. The mass range $(M_0 - M_1)$, in which a planet has three possible equilibrium configurations, is only about $0{\cdot}02\ M_E$. The total radii of different configurations of the same planet may differ by more than 100 km.

Fig. 2 shows schematically the energy of a planet as a function of its total radius ; the letters A, B, etc., refer to the same equilibrium configurations as in Fig. 1. The middle curve refers to a general mass in the interval (1). Configurations A and C with minimum energies are stable ; A has no core, and C has a large core. All configurations with small cores are of type B and are unstable ; a small disturbance will cause a transition to either A or C. It is estimated that for stability the core of a terrestrial planet must be at least 1,000 km. in radius. For this reason the phenomenon is referred to as "the instability of small planetary cores". The top and bottom curves in Fig. 2 refer to the masses M_0 and M_1, which are special cases. For these masses a maximum and a minimum in the energy diagram coalesce to give a point of inflexion, and there is only one stable configuration for each mass. For mass M_0 the stable configuration is Y', which has a core ; for mass M_1 the stable configuration is X, which has no core. The total energy of the stable configuration C with a core may be greater than or less than that of configuration A without a core, depending on the mass of the planet. The difference in energy between the two stable configurations is greatest near the ends of the critical mass range (1), that is, for masses near M_0 and M_1. The energy difference is estimated to be 10^{36}–10^{37} ergs—sufficient to remove from the surface of the planet a mass between 0·001 M_E and 0·01 M_E. Earthquakes release about 10^{26} ergs per year, and an atomic bomb about 10^{21} ergs. Thus if a planet were to undergo a transition from one configuration to another, the energy released would be enormous by ordinary standards. The transition would take minutes, or at most hours, and the energy released in this brief period would exceed that released by earthquakes during the whole of geological time. The energy liberated is always partly in the form of vibrations of the planet as a whole, but in some transitions a large fraction of the energy goes into blast waves. These blast waves, the pressure amplitude of which will be hundreds of thousands of atmospheres, will travel through the planet to the outer surface. The material on the surface will be shattered by the sudden impact of such large pressures, and fragments may fly off the planet into space. The mass of material removed may be of the order of 0·001 M_E. It is tentatively suggested that this mechanism may have been responsible for meteorites, and possibly also for the asteroids.

A planetary transition of the type envisaged will occur if the planet's mass M crosses the critical range (M_0, M_1) in either direction. The planet's mass could have changed gradually due to the aggregation of material or to the loss of light elements, depending on the mode of formation of the solar system. Alternatively, a planet could cross the critical range because of changes in the critical masses M_0 and M_1, which are functions of the internal temperature of the planet; a reasonable change in temperature could alter the critical masses by 10 per cent, and the mass of Venus is only 2 per cent greater than M_0. A planet must acquire a core if it crosses the critical mass range (1) in one direction, and it must lose its core if it crosses in the opposite direction. The core must be large at formation or disappearance since small cores are unstable; the planet must therefore undergo a transition of the type described. Venus and the earth may have undergone such transitions. The idea of an 'exploded planet' is an old one, but this mechanism makes an 'explosion' plausible for the first time. The irregular shapes and rough surfaces of the asteroids suggest that they have originated in this way. Meteorites also seem to be fragments of an 'exploded planet'. Local chemical separations, such as are apparent in stony meteorites and are necessary to account for iron meteorites, could only have occurred in a planet. In my opinion, the material of the earth below the crustal layers probably consists of ultrabasic rock with pockets of iron, such as would correspond to a representative mixture of all meteorites. Brown and Patterson[6] have recently advanced strong chemical evidence that meteorites were formed at high pressures, such as would occur in the central regions of a small planet or in the outer layers of Venus or the earth.

The present theory differs in one important respect from the old concept of an 'exploded planet'; the material removed is representative of the surface layers and not necessarily of the whole planet. Meteorites are not, of course, of the same chemical composition as surface rocks; but the transition envisaged would occur before the interior became quiescent, and so before the formation of a crust. As regards radioactivity, meteorites may be abnormal if they have originated from the surface layers of a planet; the earth's radioactivity is largely concentrated in the outer 50 km. There is circumstantial evidence that the radioactive content of meteorites is an order of magnitude greater than that of the earth as a whole.

Bullen[7] has suggested that the phenomenon under discussion may have a bearing on the origin of the moon. He reconsiders the old theory in which the moon is supposed to have been torn from the earth by a resonance tidal action of the sun. This theory has been in disrepute since Jeffreys[8] pointed out that internal damping would prevent the necessary build-up of amplitude. Bullen proposes that the function of the tidal resonance is primarily to induce a planetary transition of the type C to A in Figs. 1 and 2. He assumes that the primitive earth-moon body, due to its higher central pressure, contained within the central core another and denser core. This additional core disappeared when the moon was formed, and its disappearance released the energy necessary to separate the moon from the earth. It is highly desirable that this idea be examined quantitatively.

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